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Discrete-Time Risk Assessment of Asian Option Replication

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Québec

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The views expressed are those of the authors and do not necessarily reflect the opinions of the ministère des Finances or the National Bank of Canada.

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SUMMARY

The option pricing model proposed by Black and Scholes replicates perfectly the monetary flows of a European option. This model requires that the replication portfolio be adjusted continuously. However, this assumption is unrealistic in the context of real financial markets; the discretization of replication portfolio adjustments implies imperfect, and thus risky, reproduction of the option's monetary flows. The difference in value at expiry between the option and the replication portfolio, i.e. the tracking error, is used as a measurement of performance to assess the replication of this option. A number of earlier studies have examined European option replications; however, to our knowledge, there has been no analysis of Asian option replications. The growing utility and interest in this type of exotic option in the financial market justify such an analysis, hence the importance of knowing the risks of replicating this option.

In this study, we analyze the tracking error of discrete-time replication of an Asian option by changing five parameters that characterize the option or the dynamics of the security underlying the option. These parameters are: the frequency of adjustment of the replication portfolio, the joint volatility of the replication portfolio and the underlying security, the volatility of the underlying security while holding that of the replication portfolio constant, the option strike price and the drift in the distribution process of the underlying security. Many results are obtained, in particular the asymmetrical effects of the estimate of future volatility of the underlying security and the impact of the drift of the underlying security.

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INTRODUCTION

The major advance during the 20th century in the field of options was undoubtedly the work by Black and Scholes (1973) on the pricing of European options. They proposed, for the first time, a financial model leading to a unique price for European options. The keystone to the work of Black and Scholes is the innovative approach used to price this type of option. The financial flows of a European option can be reproduced exactly through a strategy of trading in continuous time in the security underlying the option. This approach leads to a differential equation whose solution, known for a long time, provides the price for the option. In addition to leading to a unique price for a European option, this approach provided traders who wanted to replicate such an option with an accurate method, namely the delta hedge.

However, in the real financial market, many assumptions of the Black and Scholes model are not satisfied.

Many years before the work of Black and Scholes, Mandelbrot (1963a, b; 1967) had noted the interest of studying the properties of time series of price movements in underlying securities. His results confirmed the assumptions of fat tail unconditional price distributions and change in variance over time. In addition, Mandelbrot observed that shocks are very often followed by shocks in the same direction. The assumption of constant volatility in the Black and Scholes model accordingly did not agree with the analysis of financial market data. New assumptions and new models were therefore needed.

A few months after Black and Scholes published their work, Merton (1973) extended the model for the case of non-constant determinist volatility. He showed that perfect hedging was possible if average volatility was used as the volatility of the underlying security in the Black and Scholes equation. Galai (1983) studied hedging returns by the delta of options on individual stocks traded on the Chicago Board Option Exchange (CBOE). He showed that the Black and Scholes model, with constant volatility, did not reproduce the observed average returns of the options. The conclusions of his study suggest that the factors affecting price movements of options are not included in the Black and Scholes evaluation. In particular, the idea of stochastic volatility is discussed by Galai (1983).

Many dynamic models have also been advanced to reflect non-constant volatility and the fat tail phenomenon, such as stochastic volatility models (Cox and Ross, 1976), jump models (Cox and Ross, 1975; Merton 1976) and ARCH models (Engel 1982). These assumptions regarding the dynamics of the underlying security led to the development of a substantial literature on options pricing and replication in these new dynamic frameworks.

The continuous time trading assumption of the Black and Scholes model is clearly unrealistic in a real financial market context. Accordingly, many authors looked

specifically at the impact of the discretization of trading in the underlying security. Boyle and Emmanuel (1980) demonstrate that, for European options, when the replication portfolio is adjusted discretely, the movements in the option are no longer completely correlated with those of the underlying security; accordingly, hedging is no longer perfect. In this case, the price difference between the replication portfolio and the option following a change in the price of the underlying security is convex. Replication of an option by a trader thus becomes a risky operation; a trader who replicates an option following a sale of the option is accordingly remunerated by the market for the risk he assumes. This is a partial qualitative explanation of why the implicit volatilities of traded options are greater on average than the volatilities of the underlying securities.

Boyle and Emmanuel (1980) have studied the statistical properties of the return on hedging by option replication when adjustments to the portfolio are made in discrete time. With the assumptions of the Black and Scholes model, they show that the local distribution of this return, during a short transaction period, is asymmetrical in the case of a simple European call option. Similarly, these authors show through analyses of average returns of hedged portfolios that this asymmetry leads to a bias in t-statistics.

Engle and Rosenberg (1994, 1995) consider option hedging with adjustments in discrete time in the case of dynamics with stochastic volatility. In particular, they examine the estimated values of the option gamma. The authors maintain that option hedging can be optimized using an instrument correlated to variations in volatility. For example, to hedge an option subject to a stochastic volatility parameter, another option correlated with random changes in volatility should be used in addition to the security underlying the option to be hedged. According to this process, the authors show that the delta values derived from the GARCH process are equivalent to the delta values derived from the Black and Scholes models. On the other hand, the gamma values differ significantly from the values estimated by the Black and Scholes model according to the characteristics of the option. The authors suggest that it is possible to improve hedging performances by using parameters that better represent the movements of the financial instruments used.

Bertsimas, Kogan and Lo (2000) introduce the notion of temporal granularity that characterizes the speed of convergence towards the model in continuous time in the case of discrete trading in the underlying security. The speed of convergence varies with the monetary flow and the structure of the option used for hedging; a higher gamma value of the option or a more complex distribution process of the underlying security will require a larger number of adjustment periods to reach the same level of hedge quality, if carried out over a given period. An analysis of the speed of convergence of the discrete model towards the continuous model makes it possible to study the distribution of the hedging tracking error. It also makes it possible to quantify the quality of a continuous approximation on real situations. Bertsimas, Kogan and Lo (2000) generalize the discretization study by examining the asymptotic behaviour of the tracking error for a broad range of option monetary flows and distribution processes. Accordingly, they show that the average absolute tracking error is of the order of $N^{-1/2}$, where N represents the

number of adjustments during the hedging. The authors use the local results of Boyle and Emmanuel (1980) and consider the total lifetime of the option. The distribution of the tracking error, which is locally asymmetric, becomes asymptotically symmetric.

Gobet and Temam (2001) also study the convergence of the tracking error with the increase in the number of adjustments during the hedging period. They show that the convergence rate depends very heavily on the regularity properties of the monetary flow function of the option. The rate of convergence declines with the increase in irregularity of the monetary flow of the option.

The discretization of trading in the underlying security opens the door to an analysis of the impact of transaction costs on option replication. In a context of discrete adjustments, there is a trade-off between the quality of the replication and the transaction costs involved. Accordingly, the larger the number of adjustments, the lower the uncertainty of the replication error, but the higher the transaction costs involved. The question is then to determine a level of cost at which the hedging risk is sufficiently controlled.

As shown by Leland (1985), the arbitrage relations derived from the Black and Scholes model no longer hold in the face of transaction costs. In general, including transaction costs creates limits around the theoretical price of the option within which trading profits are impossible. However, Leland (1985) stated that perfect replication was possible even when transaction costs are included; this can be achieved if, at the limit, when the time interval between adjustments tends toward zero, a hedging volatility adjusted for transaction costs is used to build the replication portfolio. However, the adjusted volatility term tends to infinity when the adjustment interval tends to zero. Moreover, Kabanov and Safarian (1997) estimated the hedging error of the Leland (1985) strategy for pricing European call options when transaction costs are included. They showed that the hedging error derived by Leland (1985) was not, at the limit, equal to zero when the level of transaction costs was constant. Kabanov and Safarian (1997) maintain that the conclusions of Leland (1985) were based on unconfirmed assumptions.

In our study, we analyze the replication of an Asian currency call option in discrete time including transaction costs. To our knowledge, no such study has been carried out previously. The dynamics of the underlying security are basically that of the Garman-Kohlhagen (1983) model, which is nothing more than a simple adaptation of the Black and Scholes model to the currency market.

Asian options are interesting because their financial flow is related to the average value of a financial asset for a given period. For example, take the case of a company that regularly buys merchandise whose price fluctuates on the market. An Asian call option means that the company will be protected against a rise in the average price of this merchandise for the given period.

These "exotic" derivatives are not traded on organized exchanges, but over the counter. In addition, the Asian options market is limited. From this standpoint, it is interesting to consider replicating such an option rather than buying it. Analysis of the replication of an Asian option is accordingly essential to a decision as to its purchase or replication.

In section 1, we briefly review Asian options, in particular the methods of pricing arithmetic Asian options. Section 2 describes the methodology used to analyze the replication of an arithmetic Asian option. We are interested in the impact of changes to five parameters on the option replication process. These parameters are: the frequency of adjustment of the replication portfolio, the joint volatility of the replication portfolio and the underlying security, the volatility of the underlying security while that of the replication portfolio is held constant, the strike price of the option and the drift of the distribution process of the underlying security. Lastly, section 3 presents the results obtained and their analysis.

SECTION 1

Review of Asian options

Asian options belong to the category of so-called "exotic" options. After Black and Scholes (1973) published their work, the number of European and American options traded on financial markets rose tremendously. However, these options alone do not satisfy all the needs of financial market players. That is why exotic options were created. They broaden the possibilities of ordinary options to meet certain specific needs of financial market players. Their structure makes it possible to create distributions of monetary flows that differ from traditional distributions. On the other hand, they have features that make them difficult to price. Indeed, it frequently happens that there is no analytical pricing solution.

An option has a specific monetary flow and properties. Options can be grouped into families with similar characteristics. There are seven general aspects that distinguish options:

- the function that defines the strike price of the option;
- the function that defines the monetary flow of the option at expiry;
- the function of the price of the underlying security;
- the strike price limits of the option;
- the function that defines the price range of the underlying security;
- the function that limits the value of the option;
- the exercise dates of the option.

The Asian option, or option on the average price of the underlying security, belongs to the family of options that depend on the path followed by the underlying security during its lifetime. It is one of the most commonly used exotic options and is traded on the over-the-counter market. Its ability to track the average price of the underlying security is the major feature that distinguishes it from ordinary options. By comparison with the latter, its market value is less volatile during its life and it is less subject to brutal variations at settlement.

The financial flow of an Asian option is defined as a function of an average of the price of the underlying security for a given period. This feature of the Asian option helps control the average purchase or sale price of a good or financial asset during a given period.

Three types of mean are generally considered for Asian options.

- Arithmetic mean:

$$A(t_N) \equiv \frac{1}{N}\sum_{i=1}^{N} S(t_i)$$

$S(t_i)$ represents the price of the underlying security when the observation is made at time t_i, where i goes from 1 to *N*. This is a flexible mean; the frequency of the t_i can be irregular, daily, weekly, monthly, etc. This type of mean can be useful in the case where a good or financial asset is traded regularly during the period, or if, in a budgetary context, certain specific dates are considered.

- Weighted arithmetic mean:

$AP(t_N) \equiv \sum_{i=1}^{N} \omega_i S(t_i)$, where the sum of the ω_i equals one.

In calculating the arithmetic mean, each observation has the same importance. However, the weights of each observation can be changed. Accordingly, the arithmetic mean is replaced with a arithmetic weighted mean. This type of mean can be particularly useful in the case where the quantity of the good or financial asset traded by the company varies over time.

- Geometric mean:

$$G(t_N) \equiv \left\{\prod_{i=1}^{N} S(t_i)\right\}^{1/N}.$$

However, this type of mean is more interesting in an academic setting than in the financial market. Note that the geometric mean can also be generalized as a weighted geometric mean.

Like ordinary options, the Asian or average option can be American or European. An American option can be exercised at any time before its expiry. The European option, however, can be exercised only at expiry. The Asian option can be a call option or a put option. We show the basic features that distinguish Asian options by taking as an example the case of an average European call option with an arithmetic mean.

Average rate Asian option. Option whose final monetary flow is given by the equation,

$$Max\left[A(t_N) - K, 0\right],$$

where K represents the strike price of the option.

Average strike price option. Option for which the strike price *K* of an ordinary European option is replaced by the average price of the underlying security during the life of the option. The final monetary flow of this type of option is given by the equation:

$$Max\left[S(t_N) - A(t_N), 0\right],$$

where $S(t_N)$ represents the price of the underlying security at expiry t_N.

Inverse average rate option. Asian option where the average rate is replaced by its inverse. This type of option is used particularly in currency market transactions. The final monetary flow of this type of option is given by:

$$Max\left[A(t_N)^{-1} - K, 0\right],$$

where $A(t_N)^{-1}$ and K are expressed in the same currency. In some cases, $A(t_N)^{-1}$ is replaced with:

$$\widetilde{A}(t_N)^{-1} = \frac{1}{\frac{1}{N}\sum_{i=1}^{N}\frac{1}{S(t_i)}}.$$

In some cases, it can be rather difficult to price an Asian option. Here we will consider the case of an arithmetic mean Asian European currency call option. The distribution model of the underlying asset is the Brownian geometric motion found in the Garman-Kohlhagen (1983) model, which is nothing more than a simple adaptation of the Black and Scholes model to the currency market. In this model, the value of this option is given by the equation:

$$C = e^{-r(T-t)} E^* \left[Max\left(\frac{1}{N}\left(\sum_{i=1}^{N} S(t_i) \right) - K, 0 \right) \right], \qquad (1)$$

where $E^*[]$: mathematical expectation under the neutral risk probability.

$dS = (r - r_f)Sdt + \sigma S\sqrt{dt}\,\varepsilon$ (stochastic distribution of neutral risk probability).

$S(t_i) = S(t_{i-1}) e^{(r - r_f - \frac{1}{2}\sigma^2)*(t_i - t_{i-1}) + \sigma\sqrt{t_i - t_{i-1}}\,\varepsilon_i}$

$S(t_i)$: CA$/US$ exchange rate at t_i.

$\varepsilon_t \sim iidN(0,1)$.

K: option strike price.

r_f: risk-free interest rate in the foreign currency (US$).

r: risk-free interest rate in the domestic currency.

T: expiry of the option.

t: time of pricing.

t_i: time when reading i of the average is taken.

σ: volatility of the underlying security.

However, this equation does not have an analytic expression. Indirect methods are therefore needed to obtain the price of the option. The Monte Carlo simulation method is a numerical procedure that is widely used to estimate this type of equation. It involves simulating the distribution process of the underlying security under the neutral risk probability. Repetition of this procedure provides an approximation of the distribution of the value of the option. The expectation under the neutral risk probability can therefore be calculated. The chief disadvantage of this method is the evaluation time needed to obtain good accuracy. This means that this procedure cannot be used efficiently in the analysis of Asian option replication.

However, the financial literature does offer approximate solutions to this equation. Vorst (1992) proposed an approximation of the value of the arithmetic mean rate option through the analytical solution of a geometric mean Asian option. However, the value of a geometric mean is always less than or equal to the value of an arithmetic mean, which introduces a bias into the estimate. The price given by the Vorst approximation is therefore a lower limit to the value of the option. To correct this bias, Vorst proposes modifying the strike price as follows:

$$K' = K - \left[E^*\left(\frac{1}{N} \sum_{i=m+1}^{N} S(t_i) \right) - E^*\left(\prod_{i=m+1}^{N} S(t_i)^{1/N} \right) \right].$$

Accordingly, the spread between the two processes is considered. Similarly, the explicit values of these expectations are given by:

$$E^*\left(\frac{1}{N}\sum_{i=m+1}^{N} S(t_i)\right) = S(t)\sum_{i=m+1}^{N} e^{(r-r_f)(t_i-t)}/N\,,$$

$$E^*\left(\prod_{i=m+1}^{N} S(t_i)^{1/N}\right) = e^{u_G+\sigma_G^2/2}\,,$$

with m mean points already recorded.

Levy (1992) proposes estimating the distribution process of the arithmetic mean rate Asian option using a normal law. He assumes that $\ln(A(t_N)) \sim N(\alpha, v^2)$ and uses the moment generating function $\ln(A(t_N))$. In addition, $E^*[A(t_N)] = e^{\alpha+v^2/2}$ and $E^*[A(t_N)^2] = e^{2\alpha+2v^2}$ have known solutions. Thus, by solving the system of equations, parameters α and v are identified:

$$\alpha = 2\ln E(A(t_N)) - \tfrac{1}{2}\ln E(A(t_N)^2)\,,$$
$$v^2 = \ln E(A(t_N)^2) - 2\ln E(A(t_N))\,.$$

The result of the Levy approximation is determined by the analytical solution of the geometric mean rate Asian option by replacing u_G by α and σ_G^2 by v^2.

Levy estimates the distribution of $A(t_N)$ with a lognormal law with the characteristic that the mean and variance correspond to the distribution to be estimated. However, a degree of uncertainty remains concerning the higher moments. For high levels of volatility (greater than 20%), the importance of these moments becomes significant and can lead to substantial bias between the real price of the option and the price given by this model.

Turnbull and Wakeman (1991) use the development of this function in series to make the necessary adjustments in view of the higher order moments. Assume $f^*(\omega)$ the conditional density function of $A(t_N)$ ($P[A(t_N) = \omega]$), by setting $a(\omega)$ as an approximate distribution, $f^*(\omega)$ can be developed as follows (Jarrow and Rudd (1992)):

$$f^*(\omega) = a(\omega) - E_1 a^{(1)}(\omega) + \tfrac{1}{2!}E_2 a^{(2)}(\omega) - \tfrac{1}{3!}E_3 a^{(3)}(\omega) + \tfrac{1}{4!}E_4 a^{(4)}(\omega) - \ldots$$

where $a^{(i)}(\omega)$ represents the derivative of order i of $a(\omega)$,

E_i represents the differences between the cumulants of the estimated and exact distributions.

For a random variable X, the first four cumulants can be expressed explicitly as follows:

$$\chi_1 = E(X),\ \chi_2 = E(X - E(X))^2,$$
$$\chi_3 = E(X - E(X))^3 \text{ and } \chi_4 = E(X - E(X))^4 - 3\chi_2.$$

Furthermore, by denoting the cumulants of order *n* associated with the estimated and exact distributions as χ_{if} and χ_{ia}, χ_{ie} represents the difference between these two values. Accordingly, the values of the coefficients E_i are:

$$E_1 = \chi_{1\varepsilon}.$$
$$E_2 = \chi_{1\varepsilon}^2 + \chi_{2\varepsilon}.$$
$$E_3 = \chi_{1\varepsilon}^3 + 3\chi_{1\varepsilon}\chi_{2\varepsilon} + \chi_{3\varepsilon}.$$
$$E_4 = \chi_{1\varepsilon}^4 + 3\chi_{2\varepsilon}^2 + 4\chi_{1\varepsilon}\chi_{3\varepsilon} + 6\chi_{1\varepsilon}^2\chi_{2\varepsilon} + \chi_{4\varepsilon}.$$

Since the first two moments of the Levy approximation correspond to the first two moments of the exact distribution, the terms E_1 and E_2 are null. Thus, the approximation of the price of the Asian option is given by the expression:

$$e^{-rT} E^* Max\left[A(t_N) - K, 0\right] =$$
$$e^{a+\frac{1}{2}v^2 - rT} N(d_1) - e^{-rT} KN(d_1 - v) - e^{-rT}\left[\tfrac{1}{3!}E_3 a^{(1)}(K) - \tfrac{1}{4!}E_4 a^{(2)}(K)\right].$$

SECTION 2

Methodology

In this section, we describe the methodology used to analyze the replication of an arithmetic mean currency (CA$/US$) Asian option. The distribution model of the underlying security is given in equation 1. The reference parameters used for the option and the dynamics of the underlying security are as follows:

Table 1

Reference parameters used for the simulation of the option replication

Starting price of the underlying security (CA$)	1.60
Strike price of the option (CA$)	1.60
Expiry of the option (years)	1
One-day interest rate in CA$	2.75% / year
One-year interest rate in CA$	3.35% / year
One-day interest rate in US$	1.75% / year
One-year interest rate in US$	2.00% / year
Volatility of the underlying security	6.25% / year
Volatility used to estimate the delta of the replication portfolio	6.25% / year
Deviation of the drift of the underlying security	0.00% / year
Frequency of adjustment of the replication portfolio	Once / day

These base parameters were determined in order to reproduce realistic data faithful to the financial market situation for 2002. Transaction costs were set at 0.05% of the value traded; they include the initial costs of purchase of the replication portfolio. When a rate of interest between one day and one year is necessary, it is obtained by linear interpolation between the one-day and one-year rates of the above table.

The replication of the option is simulated with the Monte Carlo method by applying the delta hedging method. Dupire (1998) is excellent on the application of Monte Carlo methods to finance. On the first day of trading, the net value of the replication portfolio is the value of the option on that day. This portfolio consists of a long position in the underlying security given by the delta of the option and a short position in the risk-free asset. The ensuing rebalancings are carried out so that the delta of the replication portfolio and that of the option coincide.

To make the simulation as close to reality as possible, only the replications of business days in 2002 were possible in the simulation. Six option replication frequencies were studied, namely twice a day, once a day, every second day, every third day, once a week

and once every two weeks. In the case of the twice daily adjustment, it is assumed that the adjustments were made at 10 a.m. and 2 p.m.

We studied the option replication using a Monte Carlo simulation programmed in *Visual Basic*. The congruence function used is given by the expression:

$$X_{n+1} = (X_n * a + c) Mod(2^{24})$$

where a = 214013 and c = 2531011.

The Asian call option is issued December 31, 2001 and expires on December 31, 2002. The notional value of the option is US$1. The mean of the underlying security is arithmetic monthly. The observations are recorded the last business day of each month. Accordingly, a total of 12 observations are used to calculate the mean during the year. This mean is evaluated using the following expression:

$$\frac{1}{12}\sum_{j=1}^{12} S(t_j),$$

where $S(t_j)$ represents the value of the underlying security at time t_j;

t_j represents the time of the jth observation used to calculate the mean.

A number of methods for estimating Asian options were described in section 1. The Vorst (1992) approximation was selected to estimate the value of the option during its lifetime. This function is the most efficient pricing method in view of the flexibility needed to incorporate a real calendar. Vorst (1992) also proposes an analytical solution for the approximation of the option delta. However, another method of estimating the delta was incorporated into the program in the interests of verification and reliability. It is based on an estimate of the value of a derivative by numerical differences. In addition, an estimate of the order of magnitude of the estimation error was developed to validate the results. Note that the latter function is used in the simulations.

We analyzed five parameters that characterize replication of the option. These parameters are: the frequency of adjustment of the replication portfolio, the joint volatility of the replication portfolio and the underlying security, the volatility of the underlying security while that of the replication portfolio is held constant, the strike price of the option and the drift of the distribution process of the underlying security. The Jacknife method was used to determine uncertainty on an estimated value shown in the tables of data. An excellent discussion of this method can be found in Robinson and Yang (1986).

SECTION 3

Presentation and analysis of the results

In this section, we analyze the impacts of changing five parameters that characterize the option under consideration or the dynamics of the underlying security on the replication of the option. To obtain a better interpretation of the results, we have separated the transaction cost component in our tables of results.

3.1 Impact of a change in the frequency of adjustment

The first parameter studied is the frequency of adjustment of the replication portfolio. Simulations are carried out for six different frequencies, namely: twice per business day, once per business day, once every two business days, once every three business days, one adjustment per week and lastly one adjustment every two weeks. The results obtained by Monte Carlo simulation are shown in Table 2.

Table 2

Impact of a change in the total number of adjustments to the replication portfolio

Frequency of adjustment	2 times / bus. day.	1 time / bus. day	Every 2nd bus. day	Every 3rd bus. day	1 time / week	1 time / 2 weeks
Number of Monte Carlo simulations	30 000	60 000	120 000	190 000	280 000	400 000
Total number of adjustments to the replication portfolio	492	246	123	82	52	26
Tracking error expectation (CA$)	0.000130	0.000145	0.000138	0.000122	0.000139	0.000135
Tracking error expectation uncertainty (CA$)	0.000010	0.000008	0.000008	0.000008	0.000009	0.000010
Tracking error standard deviation (CA$)	0.001790	0.001866	0.002920	0.003604	0.004901	0.006149
Tracking error standard deviation uncertainty (CA$)	0.000009	0.000006	0.000006	0.000007	0.000007	0.000007
Transaction costs expectation (CA$)	0.003672	0.003126	0.002541	0.002261	0.002008	0.001613
Transaction costs expectation uncertainty (CA$)	0.000011	0.000003	0.000002	0.000001	0.000001	0.000001

Transaction costs for the initial purchase of the replication portfolio are CA$0.000441. The value of the tracking error expectation remains stable regarding the number of adjustments made, taking into account the uncertainty of this value. In every case, this value is strictly positive allowing for uncertainty. A trader who replicates an Asian option that he sold can accordingly expect to make a profit. However, this profit will not be significant taking its standard deviation into account. The risk produced by the discretization of trading on the underlying security therefore has a cost. The standard deviation of this tracking error rises significantly as the number of adjustments to the portfolio declines, from CA$0.001790 for two adjustments per day to CA$0.006149 for one adjustment every second week. These data are illustrated in Chart 1. The longer the

interval between adjustments, the greater the chance that the spread between the replication portfolio and the value of the option increases. While the tracking error expectation is not affected by less frequent adjustments, the associated risk is clearly higher.

Transaction costs generated by adjustments to the portfolio fall significantly as the frequency of adjustment decreases, as shown in Chart 2. A decrease in the frequency of adjustment of the replication portfolio reduces the number of transactions and, by the same token, the associated costs.

The choice of frequency of replication of the option therefore rests on a trade-off between transaction costs and the risk of tracking error. The investor must choose a frequency of adjustment based on the risk he is prepared to assume and the additional amounts he is prepared to invest to reduce this risk. Chart 3 illustrates the trade-off between transaction cost expectation and the standard deviation of the tracking error that a trader who wants to replicate the option must make.

Chart 1
Tracking error standard deviation according to number of adjustments

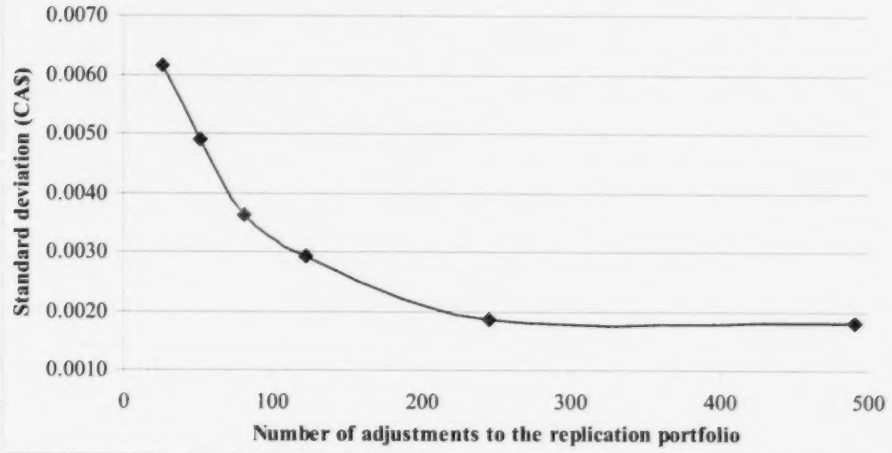
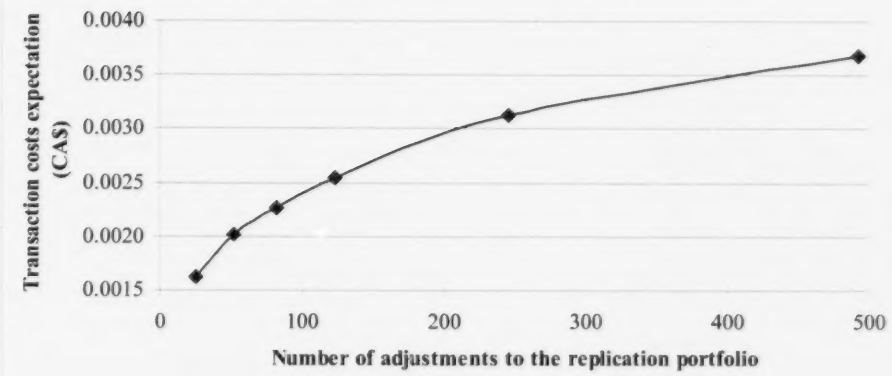


Chart 2
Transaction costs expectation according to number of adjustments



3.2 *Impact of a joint change in the implicit volatility and the underlying security*

In this section, the impact of a change in the implicit volatility and the underlying security on the replication portfolio of the option is studied. For analysis purposes, we distinguish between the volatility of the underlying security used for the Monte Carlo simulation (the distribution process of the underlying security) and the volatility used to rebalance the replication portfolio. This distinction is made to verify the impact of an over-estimate and of an under-estimate of the volatility used for option replication; these results are shown in section 3.3.

Table 3 shows the results of the simulations for a change in the joint volatility from 4.25% to 8.25%.

Table 3

Impact of a change in the joint volatility of the underlying security and the replication portfolio on the replication of the option

Joint volatility	4.25%	4.65%	5.05%	5.45%	5.85%	6.25%	6.65%	7.05%	7.45%	7.85%	8.25%
Initial value of the option (CA$)	0.020602	0.022085	0.023572	0.025062	0.026556	0.028051	0.029548	0.031047	0.032546	0.034047	0.035548
Initial delta of the option	0.574424	0.568072	0.562784	0.558329	0.554539	0.551289	0.548483	0.546045	0.543917	0.542053	0.540413
Tracking error expectation (CA$)	0.000102	0.000110	0.000118	0.000127	0.000136	0.000145	0.000154	0.000164	0.000173	0.000183	0.000192
Tracking error expectation uncertainty (CA$)	0.000005	0.000006	0.000006	0.000007	0.000007	0.000008	0.000008	0.000009	0.000009	0.000010	0.000010
Tracking error standard deviation (CA$)	0.001263	0.001384	0.001504	0.001625	0.001745	0.001866	0.001987	0.002108	0.002228	0.002349	0.002469
Tracking error standard deviation uncertainty (CA$)	0.000004	0.000005	0.000005	0.000005	0.000006	0.000006	0.000007	0.000007	0.000007	0.000008	0.000008
Transaction costs expectation (CA$)	0.003137	0.003135	0.003133	0.003130	0.003128	0.003126	0.003124	0.003122	0.003121	0.003119	0.003118
Initial transaction costs (CA$)	0.000460	0.000454	0.000450	0.000447	0.000444	0.000441	0.000439	0.000437	0.000435	0.000434	0.000432
Adjustment transaction costs expectation (CA$)	0.002677	0.002681	0.002683	0.002683	0.002684	0.002685	0.002685	0.002685	0.002686	0.002685	0.002686
Transaction costs expectation uncertainty (CA$)	0.000003	0.000003	0.000003	0.000003	0.000003	0.000003	0.000003	0.000003	0.000003	0.000003	0.000003

Chart 4
Tracking error expectation according to joint volatility

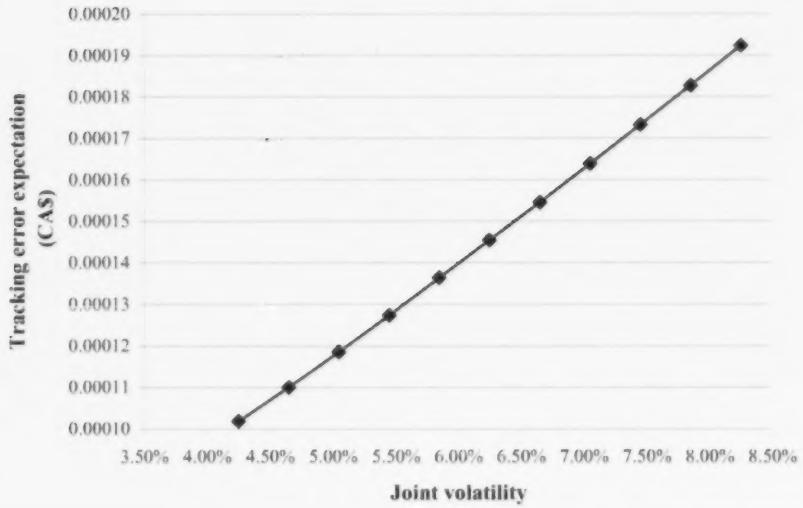


Chart 5
Tracking error standard deviation according to joint volatility

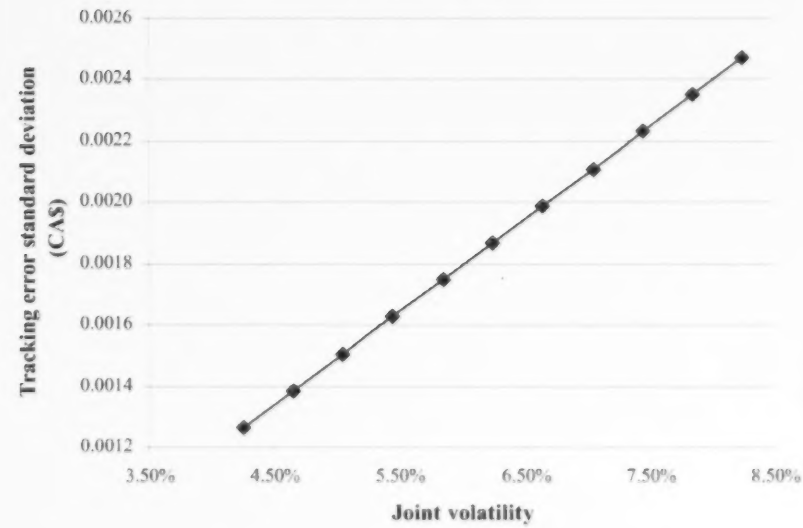


Chart 4 shows the behaviour of the tracking error expectation according to volatility. The tracking error rises linearly from CA$0.000102 for a volatility of 4.25% to CA$0.00192 for a volatility of 8.25%. The increase in the tracking error expectation is justified by the fact that an increase in volatility implies larger average movements in the value of the underlying security. Consequently, the changes in the value of the option compared to the replication portfolio are greater. Given the discretized adjustments to the replication portfolio, the latter is less effective in following the path of the value of the option when shocks are major. The changes in the delta portion to be held are larger. Similarly, the risk associated with replication is higher because of the replication portfolio's lesser ability to accurately track the value of the option. As illustrated in Chart 5, we see that the tracking error rises as joint volatility increases. Accordingly, the values of the distribution of the tracking error are more scattered and extreme as volatility increases, generating greater uncertainty.

Transaction costs remain fairly stable even when joint volatility rises significantly; the change is less than 0.7% between joint volatilities running from 4.25% to 8.25%. This small decline is essentially attributable to initial transaction costs. For the cases studied, the greater the joint volatility, the smaller the initial delta of the option to be replicated. This generates less transaction costs when the replication portfolio is initially formed.

3.3 Impact of a change in the volatility of the underlying security

In this section, we give the results of the analysis of the impact of a change in the volatility of the underlying security on the replication of the option. Transaction costs on the initial purchase of the replication portfolio are CA$0.000441. Unlike the analysis in the preceding section, the implicit volatility of the replication portfolio differs from that of the underlying security; it is kept constant at the reference value, i.e. 6.25%. Table 4 shows the results of the Monte Carlo simulations for a change in the volatility of the underlying security from 4.25% to 8.25%.

Table 4

Impact of a change in volatility of the underlying security

Volatility of the underlying security	4.25%	4.65%	5.05%	5.45%	5.85%	6.25%	6.65%	7.05%	7.45%	7.85%	8.25%
Tracking error expectation (CA$)	-0.007562	-0.006029	-0.004491	-0.002949	-0.001403	0.000145	0.001696	0.003248	0.004802	0.006357	0.007913
Tracking error expectation uncertainty (CA$)	0.000011	0.000010	0.000009	0.000008	0.000007	0.000008	0.000009	0.000010	0.000012	0.000015	0.000017
Tracking error standard deviation (CA$)	0.002646	0.002387	0.002131	0.001920	0.001813	0.001866	0.002101	0.002491	0.002997	0.003585	0.004234
Tracking error standard deviation uncertainty (CA$)	0.000006	0.000006	0.000006	0.000005	0.000006	0.000006	0.000007	0.000008	0.000010	0.000011	0.000013
Transaction costs expectation (CA$)	0.002709	0.002804	0.002892	0.002975	0.003053	0.003126	0.003195	0.003260	0.003321	0.003379	0.003434
Transaction costs expectation uncertainty (CA$)	0.000002	0.000002	0.000003	0.000003	0.000003	0.000003	0.000004	0.000004	0.000004	0.000004	0.000004

Chart 6
Tracking error expectation according to the volatility of the underlying security

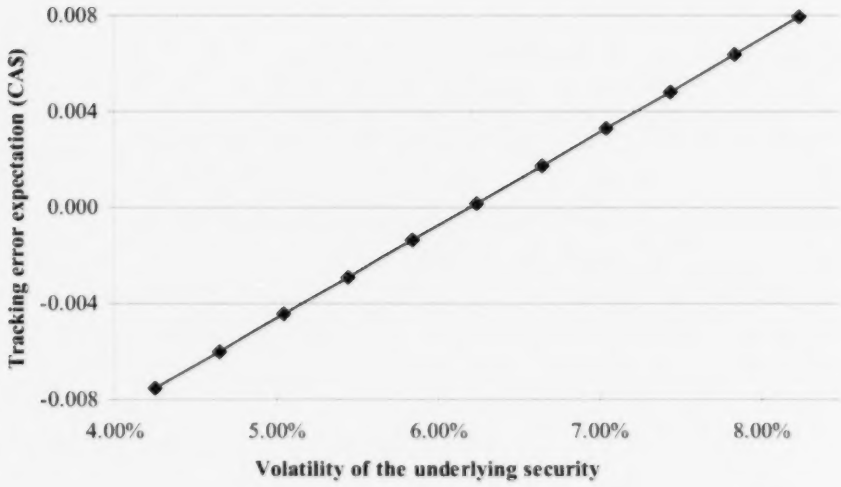
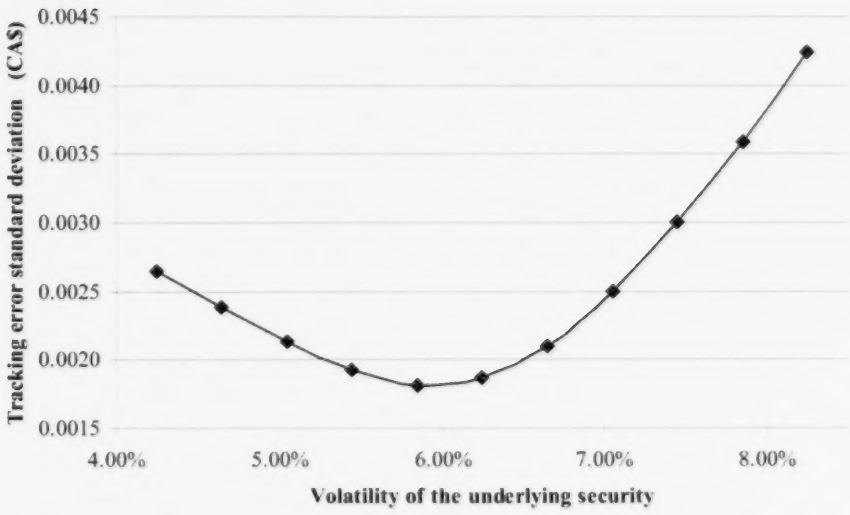


Chart 7
Tracking error standard deviation according to the volatility of the underlying security



The change in volatility of the underlying security implies a change in the size of its movements. The tracking error expectation, illustrated in Chart 6, grows with volatility. In the case where replication volatility is greater than the volatility of the underlying security, the trader who sells and replicates the option according to replication volatility makes profits; the value of the replication portfolio will, on average, be greater than that of the monetary flow of the option. However, if the replication volatility is under-estimated compared with real market volatility, the holder of the replication portfolio will, on average, suffer negative profits that increase with the spread between the two volatilities.

We now turn to the standard deviation of the tracking error distribution. The objective of an investor who replicates an option is to obtain the lowest uncertainty possible associated with the replication. Accordingly, it is in the investor's interest to select replication parameters that allow him to minimize this risk.

The standard deviations of the monetary flow and of the replication portfolio become greater with the volatility of the underlying security. When the volatility of the underlying security is less than the volatility used for replication, the variance of the monetary flow is greater than that of the replication portfolio. Similarly, when the volatility of the underlying portfolio is greater than the replication volatility, the variance of the replication portfolio is greater than that of the monetary flow. Given the very strong correlation of the two random variables (very close to 1), there is a point that minimizes the uncertainty of replication.

The tracking error data show that uncertainty is significantly minimal for volatility of the underlying security equal to 5.85% according to the simulation results. Around this value, uncertainty rises non-linearly for higher or lower volatility values of the underlying security. The tracking error standard deviation according to the volatility of the underlying security, shown in Chart 7, illustrates this behaviour. This important result implies that it is preferable for an investor who replicates the option under study to choose a replication portfolio volatility slightly greater (6.25% compared with 5.85%) than the real volatility of the underlying security. A slight over-estimation of market volatility in building the replication portfolio will enable him to minimize the risk associated with replication. In addition, such positioning implies an expected tracking error value that is lower and negative compared to a choice of portfolio volatility that is identical to market volatility. This situation is reflected in expected gains for an investor who replicates the option. In the same vein, such a choice implies lower expected transaction costs and a smaller initial investment. Note that this investment corresponds to the value of the option when it is issued.

Transaction costs grow significantly with the level of volatility of the underlying security, rising from CA$0.0027091 for volatility of the underlying security of 4.25% to CA$0.003434 for volatility of 8.25%. Linked with the change in the dispersion of the value of the replication portfolio, transaction costs are directly influenced by changes to the evaluation of the delta of the option. Since the dispersion of the delta values of the option grows with the volatility of the underlying security, the transaction costs implied at each adjustment also grow given the wider spreads between the deltas. Similarly, unlike the preceding section, the initial delta value used to create the replication portfolio is constant. As we saw in the preceding sub-section, the element that contributed to slightly reducing transaction costs as volatility increases is not present here, given the constant volatility used for the Vorst evaluation. The growth in the dispersion of delta values is accordingly reflected in the growth in transaction costs.

3.4 Impact of a change in the strike price of the option

Table 5 shows the results of simulations for a change in the strike price from CA$1.40 (in-the-money option) to CA$1.80 (out-of-the-money option).

Table 5

Impact of a change in the strike price of the option on its replication

Strike price (CA$)	1.40	1.44	1.48	1.52	1.56	1.60	1.64	1.68	1.72	1.76	1.80
Initial value of the option (Vorst) (CA$)	0.202940	0.164053	0.125392	0.087916	0.054210	0.028051	0.011691	0.003814	0.000959	0.000185	0.000027
Initial delta of the option (Vorst)	0.977973	0.976463	0.964974	0.914201	0.778504	0.551289	0.305113	0.127397	0.039596	0.009162	0.001590
Tracking error expectation (CA$)	0.000039	0.000036	0.000033	0.000042	0.000081	0.000145	0.000162	0.000118	0.000058	0.000021	0.000005
Tracking error expectation uncertainty (CA$)	0.000000	0.000000	0.000001	0.000003	0.000006	0.000008	0.000007	0.000005	0.000003	0.000001	0.000001
Tracking error standard deviation (CA$)	0.000052	0.000115	0.000348	0.000836	0.001463	0.001866	0.001751	0.001240	0.000702	0.000330	0.000132
Tracking error standard deviation uncertainty (CA$)	0.000001	0.000003	0.000004	0.000005	0.000006	0.000006	0.000006	0.000006	0.000006	0.000006	0.000005
Transaction costs expectation (CA$)	0.001583	0.001605	0.001740	0.002164	0.002824	0.003126	0.002561	0.001486	0.000613	0.000183	0.000040
Initial transaction costs (CA$)	0.000782	0.000781	0.000772	0.000731	0.000623	0.000441	0.000244	0.000102	0.000032	0.000007	0.000001
Transaction costs expectation uncertainty (CA$)	< 0.0000001	< 0.0000001	0.000001	0.000003	0.000003	0.000003	0.000004	0.000005	0.000003	0.000002	0.000001

Chart 8
Tracking error expectation according to the strike price of the option

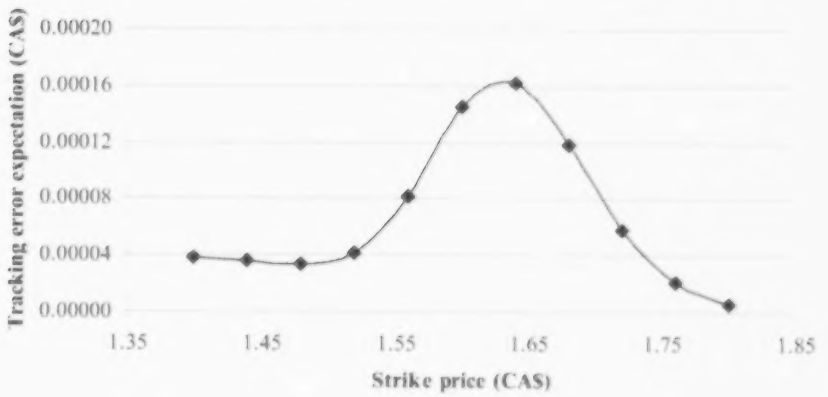


Chart 9
Tracking error standard deviation according to the strike price of the option

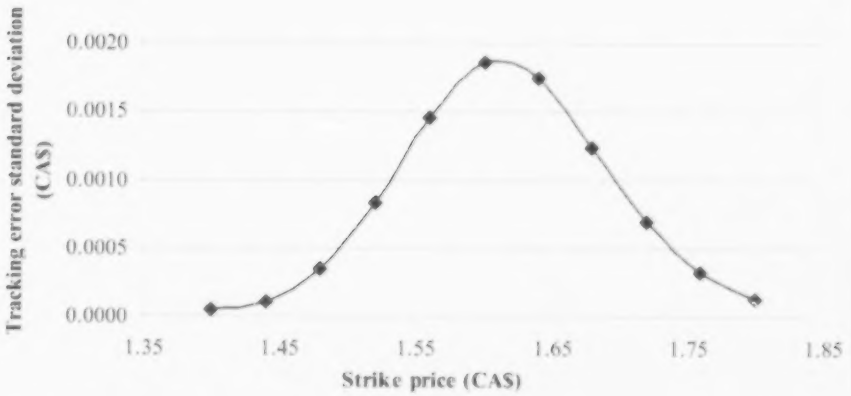
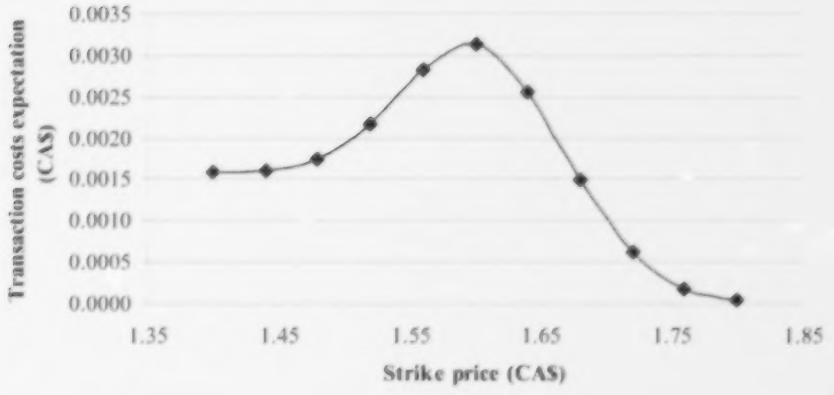


Chart 10
Transaction costs expectation according to the strike price of the option



The tracking error expected value, illustrated in Chart 8, is a minimum when the option is deeply in the money or deeply out of the money. In addition, this value is a maximum when the option approaches the at-the-money value. Intuitively, movements in the value of the underlying security have much less impact on the value of delta for an option far from the at-the-money value. The initial values of delta, for the various simulated points, present maximum spreads from one point to another for an option near the at-the-money value. In this case, the movements of the delta portion of the replication portfolio, from one adjustment to another, are larger. Accordingly, this portfolio is less effective in tracking the exact value of the Asian option, implying a greater average tracking error.

The risk level associated with the replication of the option, the tracking error standard deviation, is also significantly maximal for an option near the at-the-money value. Chart 9 shows this behaviour. Following the explanation in the preceding paragraph, the delta values of an option at the at-the-money value are more sensitive and exhibit larger movements, for equivalent changes in the value of the underlying security, than the delta values of an option that is far from the at-the-money value. The replication portfolio of an option at the at-the-money value tracks the exact value of the option less well, so the risk associated with the replication is also greater.

Lastly, the transaction costs associated with replication, shown in Chart 10, are also significantly maximal when the option is at the at-the-money value given the larger movements in the delta value. However, it should be noted that an option that is deeply out of the money implies practically zero cost while an option that is deeply in the money implies slightly higher costs. While the delta value of an option that is deeply out of the money implies holding a very small portion of the underlying security, the replication of an

option that is deeply in the money implies the initial purchase of a large portion of the underlying security. The transaction costs implied by this purchase account for the difference between the amount of transaction costs of options that are deeply in the money compared with options that are deeply out of the money.

3.5 Impact of introducing a drift deviation parameter

The Black and Scholes European options pricing model implies a basic property: the pricing process considers only the volatility term of the underlying security, regardless of the drift parameter. Accordingly, Black and Scholes pricing of the value of an option will be identical for two underlying securities with similar characteristics but different drift parameters. Continuous-time pricing implies that the volatility of an underlying security is sufficient to determine the arbitrage price of a contingent security. The assumption of continuous-time trading was lifted for reasons that have already been mentioned. This section studies the effects of introducing a drift parameter in the distribution equation of the underlying security in a discontinuous context. Taking distribution equation 1 and adding the drift term, the equation used for the simulations becomes:

$$S(t_i) = S(t_{i-1})e^{((r-r_f+dérive)-\frac{1}{2}\sigma^2)*(t_i-t_{i-1})+\sigma\sqrt{t_i-t_{i-1}}\epsilon_i}$$

The simulations are carried out for values of the drift parameter varying from -2% to 2% and the results are shown in Table 6.

Table 6

Impact of introducing a drift deviation parameter

Drift	-2.00%	-1.60%	-1.20%	-0.80%	-0.40%	0.00%	0.40%	0.80%	1.20%	1.60%	2.00%
Tracking error expectation (CA$)	0.000182	0.000175	0.000167	0.000159	0.000152	0.000145	0.000137627	0.000132	0.000126	0.000121	0.000117
Tracking error expectation uncertainty (CA$)	0.000008	0.000008	0.000008	0.000008	0.000008	0.000008	7.60378E-06	0.000008	0.000008	0.000008	0.000008
Tracking error standard deviation (CA$)	0.001878	0.001877	0.001875	0.001873	0.001870	0.001866	0.001862539	0.001857	0.001852	0.001845	0.001839
Tracking error standard deviation uncertainty (CA$)	0.000006	0.000006	0.000006	0.000006	0.000006	0.000006	6.13603E-06	0.000006	0.000006	0.000006	0.000006
Transaction costs expectation (CA$)	0.003081	0.003093	0.003103	0.003112	0.003120	0.003126	0.003130498	0.003133	0.003134	0.003134	0.003131
Transaction costs expectation uncertainty (CA$)	0.000004	0.000004	0.000004	0.000003	0.000003	0.000003	3.42784E-06	0.000003	0.000003	0.000003	0.000003

Chart 11
Tracking error expectation according to the drift parameter

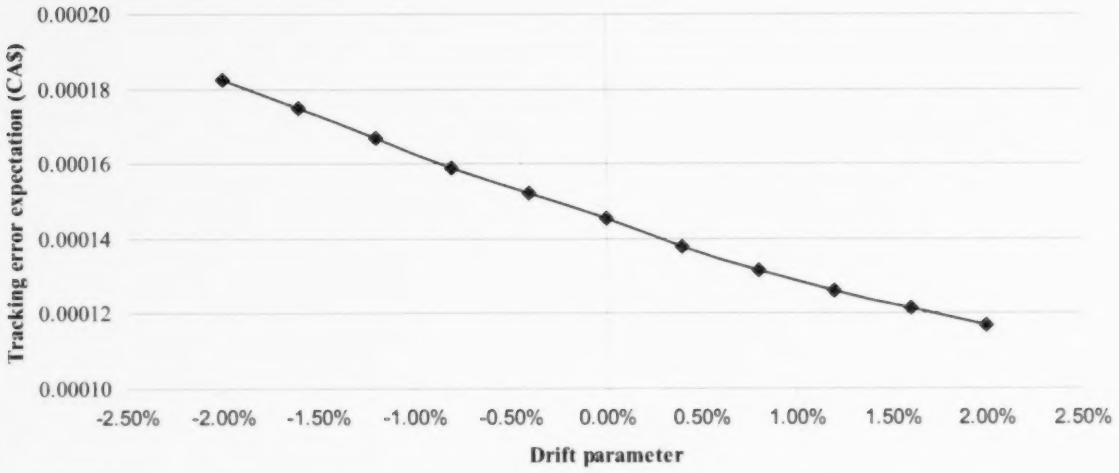


Chart 12
Tracking error standard deviation according to the drift parameter

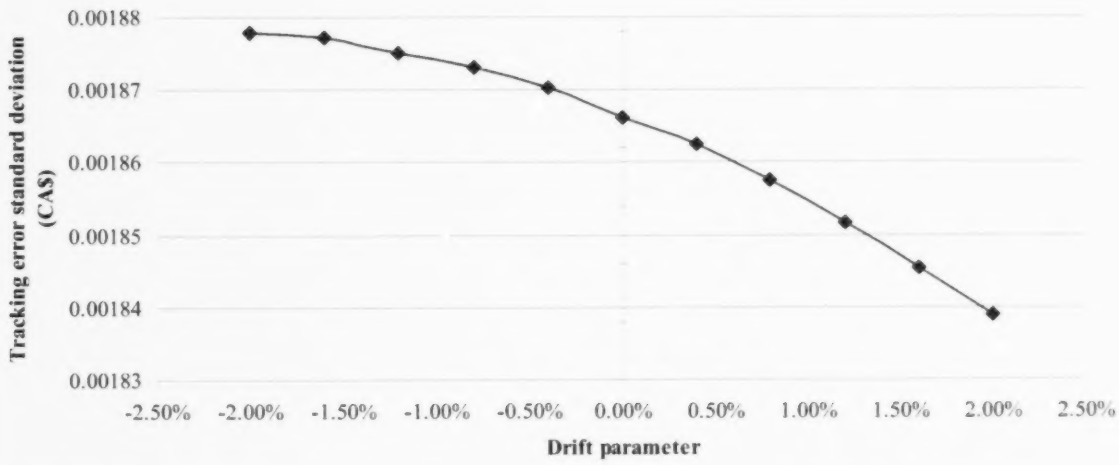
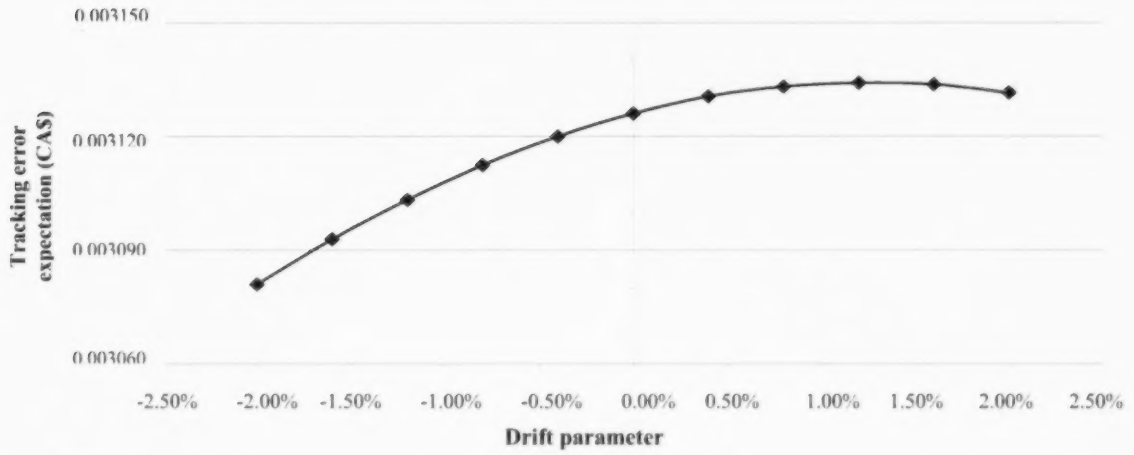


Chart 13
Transaction cost expectation
according to the drift parameter



The transaction costs for the initial purchase of the replication portfolio are CA$0.000441. The standard deviation of the simulation increases significantly with the increase in drift, from CA$0.033474 for drift of -2% to CA$0.045774 for drift of 2%. Intuitively, given that the basic parameters under study present an option at the money, an increasingly negative drift implies, on average, a growing number of negative monetary flow paths. These paths are truncated at zero. Accordingly, the larger the number of negative monetary flow paths, the more the dispersion of the expected value of the option's monetary flow declines.

The replication tracking error expectation decreases as the drift parameter grows, falling from CA$0.000182 for drift of -2% to CA$0.000117 for drift of 2%. This behaviour is shown in Chart 11. The expected tracking error falls. The correlation between the replication portfolio and the monetary flow rises with drift.

The uncertainty related to the replication error, shown in Chart 12, also declines as drift rises, from CA$0.001878 for a drift of -2% to CA$0.001839 for a drift of 2%. Once again, these differences are not important from one variation to another. The difference becomes appreciable for a change of 3.2% in drift. The variance of the monetary flow of the option and the variance of the value of the replication portfolio increase with the drift parameter.

Transaction costs rise with drift. However, this increase is not appreciable from one estimated point to another; it does become appreciable for a change in drift of 0.8%. As we have already mentioned, transaction costs depend on changes in the delta value estimated by Vorst and in the value of the underlying security. On the one hand, the more the drift parameter grows, the higher, on average, the value of the underlying security, which implies an increase in costs at each adjustment. On the other, as drift grows in absolute value, the dispersion of the value of the underlying security increases. This increase in dispersion implies that the movements in the delta value between each adjustment are larger when drift is increasingly negative (for negative drift) or increasingly positive (for positive drift).

3.6 Summary of results

Table 7 summarizes the results obtained in sections 3.1 to 3.5.

Table 7

Summary of the impact of the increase in parameters on the option replication

Parameter / Impact	Tracking error expectation	Tracking error volatility	Transaction costs expectation
Number of adjustments	Stable	Decrease	Increase
Joint volatility	Increase	Increase	Small decrease
Volatility of underlying security	Increase	Convex function	Increase
Option strike price	Function with maximum	Function with maximum	Function with maximum
Drift of underlying security	Decrease	Decrease	Increase

CONCLUSION

The replication of an Asian currency call option was analyzed using a Monte Carlo simulation process. We studied the impact of changing five parameters that characterize the option or the dynamics of the underlying security on the replication of the option. This leads to the analysis of a risk-return relation stemming from the replication of the option. Thus, depending on the interests of the financial market player wishing to replicate the option, an optimum positioning can be identified.

The tracking error expectation is constant according to the frequency of adjustment. However, an investor will choose a higher adjustment frequency to reduce the uncertainty associated with tracking error. This declines significantly as the frequency of adjustment of the replication portfolio rises. At the same time, this choice implies higher transaction costs. The investor must therefore identify a compromise between the uncertainty associated with replication and the implied transaction costs.

The change in the level of joint volatility of the replication portfolio and of the underlying security (volatility of the underlying security equivalent to the volatility of the replication portfolio) is the second parameter considered. The tracking error expectation, as well as the associated uncertainty, also rises as volatility increases. The investor who replicates the option therefore holds a riskier and more costly position (higher tracking error) when volatility is high. On the other hand, given that transaction costs decline slightly when volatility rises, the investor may have lower transaction costs if volatility is high.

A change in the volatility of the underlying security, while keeping that of the replication portfolio constant, is the third point of analysis. The expected tracking error also rises as the volatility of the underlying security increases. However, the uncertainty associated with the replication is minimal for volatility of the underlying security that is slightly less than the volatility of the replication. Accordingly, an investor seeking to minimize risk associated with replication will choose a replication volatility slightly higher than the volatility of the underlying security (market volatility). In this case, such a position implies a slightly negative expected tracking error. Therefore, the value of the replication portfolio according to this choice will, in general, be higher than the value of the final monetary flow of the option. In addition, since the transaction costs associated with replication grow significantly as the volatility of the underlying security increases, the decision to slightly overestimate the volatility of the market implies a decrease in transaction costs.

The fourth point considered in the analysis is a change in the strike price of the replicated option. The expected tracking error is a maximum when the option is at the money. It declines as the option moves deeply in the money or deeply out of the money. The uncertainty associated with the tracking error exhibits similar behaviour and is a maximum when the option is at the money. Transaction costs also fall significantly as the option moves farther away from the at-the-money value. However, they are minimal when the option is deeply out of the money; an option that is deeply in the money implies the purchase of large portion of underlying securities and accordingly higher transaction costs. The investor seeking to minimize tracking error, the risks associated with replication and transaction costs will therefore choose an option that is not at the money and preferably out of the money. However, it is clear that the primary reason for exercising the replication is to hedge expenses resulting from changes in the exchange rate. The choice of the at-the-money value must therefore satisfy hedging objectives above all. On the other hand, as far as possible, an efficient positioning implies lower replication costs and uncertainty.

The introduction of a drift deviation parameter in the distribution equation of the underlying security is the last point analyzed. The expected tracking error and the risk associated with it fall as drift increases. Accordingly it is very interesting for an investor to replicate an option whose underlying security is characterized by a positive drift deviation. Such a situation means that the replication is less risky and the final value of the replication portfolio is closer to the final monetary flow of the option. Lastly, it should be noted that transaction costs rise as drift increases.

Obviously, the basic structure considered in this study concentrates chiefly on the analysis of the impact of trading in discrete time on the replication of an Asian option. The literature developed in order to minimize transaction costs associated with replication is a very promising approach from the standpoint of extending the study. It would be extremely interesting to introduce alternative adjustment strategies in order to identify their real contribution to the process of optimizing the costs of Asian option replication. Accordingly, modeling of the super replication, adjustments based on the movement of the underlying security, adjustments based on the movement of the delta of the option and multi-period adjustments would be interesting developments. Our analysis is based on the use of constant volatility during the lifetime of the option. This assumption could be dropped in order to introduce volatility models that evolve over time. A stochastic model of volatility could be another interesting approach to explore.

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The option pricing model proposed by Black and Scholes replicates perfectly the monetary flows of a European option. This model requires that the replication portfolio be adjusted continuously. However, this assumption is unrealistic in the context of real financial markets; the discretization of replication portfolio adjustments implies imperfect, and thus risky, reproduction of the option's monetary flows. The difference in value at expiry between the option and the replication portfolio, i.e. the tracking error, is used as a measurement of performance to assess the replication of this option. A number of earlier studies have examined European option replications; however, to our knowledge, there has been no analysis of Asian option replications. The growing utility and interest in this type of exotic option in the financial market justify such an analysis, hence the importance of knowing the risks of replicating this option.

In this study, we analyze the tracking error of discrete-time replication of an Asian option by changing five parameters that characterize the option or the dynamics of the security underlying the option. These parameters are: the frequency of adjustment of the replication portfolio, the joint volatility of the replication portfolio and the underlying security, the volatility of the underlying security while holding that of the replication portfolio constant, the option strike price and the drift in the distribution process of the underlying security. Many results are obtained, in particular the asymmetrical effects of the estimate of future volatility of the underlying security and the impact of the drift of the underlying security.

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